Multi-view Geometry

Readings: FP10

March 11, 2006

Multi-view Geometry and 3D

- **We have 2 eyes , thus we see 3D**
- **Using multiple views allows inference of the 3rd dimension**

How to see in 3D

(Using geometry…)

- **Find corresponding features**
- **Triangulate and reconstruct depth**

Correspondence

Given a point in one image, find the point in a second image of the same 3-D location.

One of the hardest vision problems!

Last Lecture: Algorithms for (quickly) estimating best correspondences.

Now: Where do we search? What are the constraints between images of 3-D points in multiple views?

Outline

- Multi-view geometry
- Epipolar constraint
- Essential matrix
- Fundamental matrix
- Trifocal tensor

Multi-view Geometry

Relate

Multi-view Geometry

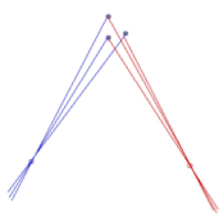
Relate
- 3-D points

Multi-view Geometry

Relate
- 3-D points
- Camera centers

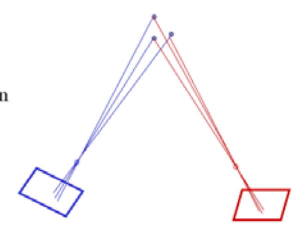
Multi-view Geometry

Relate
- 3-D points
- Camera centers
- Camera orientation



Multi-view Geometry

Relate
- 3-D points
- Camera centers
- Camera orientation
- Camera intrinsics



Multi-view Geometry

Relate
- 3-D points
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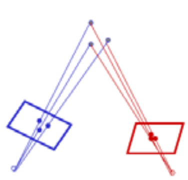
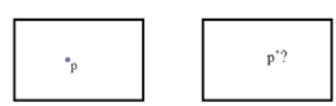
- Camera intrinsics



Stereo Constraints

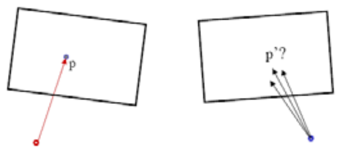


Given p in left image, where can corresponding point p' be?

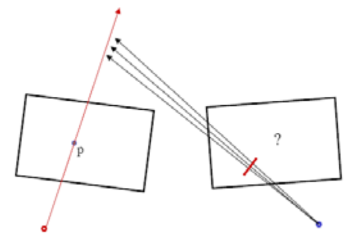
Could be anywhere! Might not be same scene!
… Assume pair of pinhole views of static scene:

Stereo Constraints

Given p in left image, where can p' be?



Epipolar Line



Epipolar Constraint

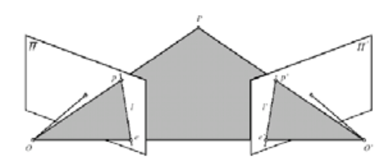


FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

All epipolar lines contain epipole, the image of other camera center.

Epipolar Constraint

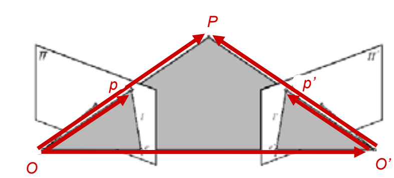
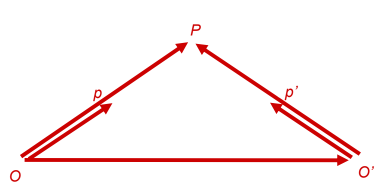


FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

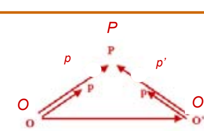
All epipolar lines contain epipole, the image of other camera center.

From Geometry to Algebra



The epipolar constraint: these vectors are coplanar:

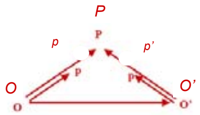
$$\overrightarrow{Op}\cdot[\overrightarrow{OO'}\times\overrightarrow{O'p'}]=0$$



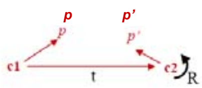
$$\overrightarrow{Op}\cdot\left[\overrightarrow{OO'}\times\overrightarrow{O'p'}\right]=0$$

$$\boldsymbol{p}\cdot\left[\boldsymbol{t}\times\left(\boldsymbol{t}+\mathcal{R}\,\boldsymbol{p}'\right)\right]=0$$

$$\boldsymbol{p}\cdot\left[\boldsymbol{t}\times\left(\mathcal{R}\,\boldsymbol{p}'\right)\right]=0$$



$$\overrightarrow{Op}\cdot\left[\overrightarrow{OO'}\times\overrightarrow{O'p'}\right]=0$$



p,p' are image coordinates of P in c1 and c2…

c2 is related to c1 by rotation R and translation t

$$p\cdot\left[t\times\left(t+\mathcal{R}\,p'\right)\right]=0$$

$$p\cdot\left[t\times\left(\mathcal{R}\,p'\right)\right]=0$$

Matrix Form

$$p\cdot[t\times(\mathcal{R}p')]=0$$

Linear constraint, should be able to express as matrix equation…

Review: Matrix Form of the Cross Product

The vector cross product also acts on two vectors and returns a third vector. Geometrically, this new vector is constructed such that its projection onto either of the two input vectors is zero.

$$\vec{a}\times\vec{b}=\begin{bmatrix}a_yb_z-a_zb_y\\a_zb_x-a_xb_z\\a_xb_y-a_yb_x\end{bmatrix}$$

$$\vec{a}\times\vec{b}=\begin{bmatrix}0&-a_z&a_y\\a_z&0&-a_x\\-a_y&a_x&0\end{bmatrix}\begin{bmatrix}b_x\\b_y\\b_z\end{bmatrix}=\vec{c}\qquad\begin{matrix}\vec{a}\cdot\vec{c}=0\\\vec{b}\cdot\vec{c}=0\end{matrix}$$

Review: Matrix Form of the Cross Product

$$\vec{a}\times\vec{b}=\begin{bmatrix}0&-a_z&a_y\\a_z&0&-a_x\\-a_y&a_x&0\end{bmatrix}\begin{bmatrix}b_x\\b_y\\b_z\end{bmatrix}=\vec{c}\qquad\begin{matrix}\vec{a}\cdot\vec{c}=0\\\vec{b}\cdot\vec{c}=0\end{matrix}$$

$$[a_x]=\begin{bmatrix}0&-a_z&a_y\\a_z&0&-a_x\\-a_y&a_x&0\end{bmatrix}$$

Review: Matrix Form of the Cross Product

$$\vec{a}\times\vec{b}=\begin{bmatrix}0&-a_z&a_y\\a_z&0&-a_x\\-a_y&a_x&0\end{bmatrix}\begin{bmatrix}b_x\\b_y\\b_z\end{bmatrix}=\vec{c}\qquad\begin{matrix}\vec{a}\cdot\vec{c}=0\\\vec{b}\cdot\vec{c}=0\end{matrix}$$

$$[a_x]=\begin{bmatrix}0&-a_z&a_y\\a_z&0&-a_x\\-a_y&a_x&0\end{bmatrix}$$

$$\boxed{\vec{a}\times\vec{b}=[a_x]\vec{b}}$$

Matrix Form

$$p\cdot[t\times(\mathcal{R}p')]=0$$

Matrix Form

$$\boldsymbol{p}\cdot[\boldsymbol{t}\times(\mathcal{R}\boldsymbol{p}')]=0$$

$$\vec{a}\times\vec{b}=[a_x]\vec{b}$$

$$p^T[t_x]\Re p'=0$$

$$\varepsilon=[t_x]\Re$$

$$\boldsymbol{p}^T\mathcal{E}\boldsymbol{p}'=0$$

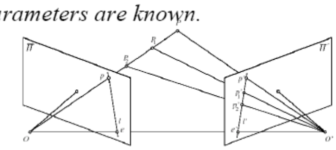
Essential Matrix

Matrix that relates image of point in one camera to a second camera, given translation and rotation.

5 independent parameters (up to scale)

Assumes intrinsic parameters are known.

$$\varepsilon=[t_x]\Re$$



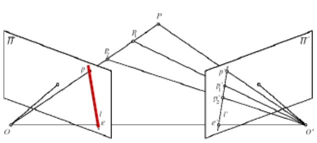
$$\boldsymbol{p}^T\mathcal{E}\boldsymbol{p}'=0$$

$$\vec{a}\times\vec{b}=[a_x]\vec{b}$$

Epipolar Line Constraint

$\mathcal{E}p'$ is the epipolar line corresponding to p' in the left camera.

$$au+bv+c=0$$



$\mathcal{E}p'$

$$p=(u,v,1)^T$$
$$l=(a,b,c)^T$$
$$l\cdot p=0$$

$$\boldsymbol{p}^T\mathcal{E}\boldsymbol{p}'=0$$

$$\mathcal{E}p'\cdot p=0$$

Essential Matrix – Instantaneous case

- **For small motion given translation and rotation velocity:**

$$\boldsymbol{t}=\delta t\,\boldsymbol{v},$$
$$\mathcal{R}=\mathrm{I}'+\delta t\,[\boldsymbol{\omega}_\times]$$
$$\boldsymbol{p}'=\boldsymbol{p}+\delta t\,\dot{\boldsymbol{p}}.$$

$$\boldsymbol{p}^T\mathcal{E}\boldsymbol{p}'=0 \qquad p^T[t_x]\Re p'=0$$

$$\boldsymbol{p}^T\underbrace{[\boldsymbol{v}_\times](\mathrm{I}+\delta t\,[\boldsymbol{\omega}_\times])}_{\mathcal{E}}(\boldsymbol{p}+\delta t\,\dot{\boldsymbol{p}})=0$$

Focus of Expansion for Translating Camera

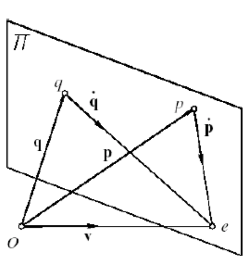
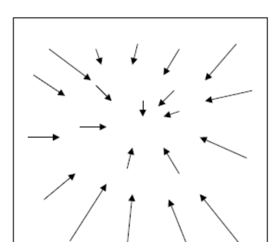


FIGURE 11.3: Focus of expansion: under pure translation, the motion field at every point in the image points toward the focus of expansion.

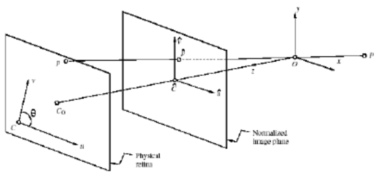
FOE for Translating Camera



What if calibration is unknown?

Recall calibration eqn:

$$\boldsymbol{p} = \mathcal{K}\hat{\boldsymbol{p}}, \quad \text{where} \quad \boldsymbol{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \quad \text{and} \quad \mathcal{K} \stackrel{\text{def}}{=} \begin{pmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \frac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}.$$



Fundamental Matrix

Essential matrix for points on normalized image plane,

$$\hat{p}^T \mathcal{E} \hat{p}' = 0$$

assume unknown calibration matrix:

$$p = K\hat{p}$$

yields:

$$\boxed{\boldsymbol{p}^T \mathcal{F} \boldsymbol{p}' = 0} \qquad \mathcal{F} = \mathcal{K}^{-T} \mathcal{E} \mathcal{K}'^{-1}$$

Estimating the Fundamental Matrix

$$\boldsymbol{p}^T \mathcal{F} \boldsymbol{p}' = 0$$

Each point correspondence can be expressed as a single linear equation

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

Estimating the Fundamental Matrix

$$\boldsymbol{p}^T \mathcal{F} \boldsymbol{p}' = 0$$

Each point correspondence can be expressed as a single linear equation

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Leftrightarrow (uu', uv', u, vu', vv', v, u', v', 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

8 Point Algorithm

8 corresponding points, 8 equations.

$$\begin{pmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' \\ u_3u_3' & u_3v_3' & u_3 & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' \\ u_4u_4' & u_4v_4' & u_4 & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' \\ u_5u_5' & u_5v_5' & u_5 & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' \\ u_6u_6' & u_6v_6' & u_6 & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' \\ u_7u_7' & u_7v_7' & u_7 & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' \\ u_8u_8' & u_8v_8' & u_8 & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8' \end{pmatrix}\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Invert and solve for $\mathcal{F}$.

(Use more points if available; find least-squares solution to minimize $\sum_{i=1}^{n}(\boldsymbol{p}_i^T \mathcal{F} \boldsymbol{p}_i')^2$)

8 Point Algorithm

$$\boldsymbol{p}^T \mathcal{F} \boldsymbol{p}' = 0$$

is $\mathcal{F}$ (or $\mathcal{E}$) full rank?

No…singular with rank=2.
Has zero eigenvalue corresponding to epipole.

$$\mathcal{F}^T e = 0$$

(Note that $\mathcal{E}$ has two equal singlar values [Huang and Faugeras 1989])

Improved 8 Point Algorithm

Enforce rank 2 constraint!

(Also pay attention to numerical conditioning…)

Hartley 1995: use SVD.

1. Transform to centered and scaled coordinates
2. Form least-squares estimate of F
3. Set smallest singular value to zero.

Zhengyou Zhang
Determining the Epipolar Geometry and its Uncertainty: A Review
www.cs.unr.edu/~mircea/Courses/ cs790E/Lectures/zhang2.ppt

Normalizing the Input Data

- Directly use the pixel coordinates produces bad result
- Normalization method is quite necessary
- Isotropic scaling of the input data:
  - Points are translated to have their centroid at the origin
  - The coordinates are scaled isotropically so that the average distance from the origin to these points is equal to $\sqrt{2}$.

Zhengyou Zhang 26
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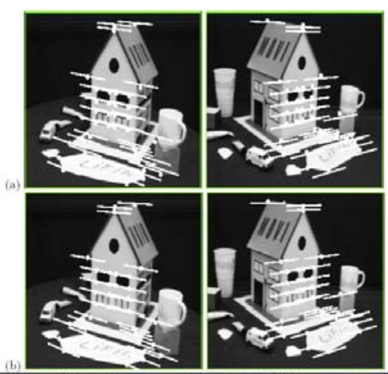
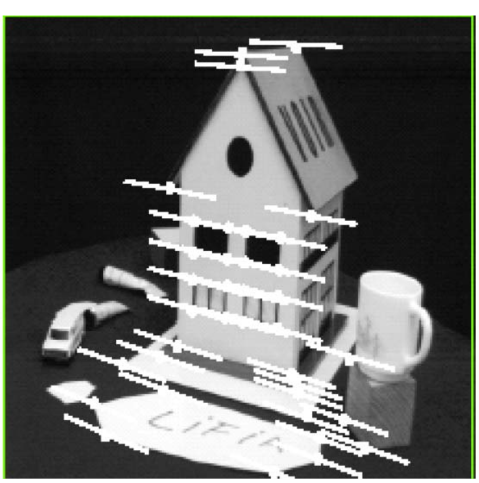
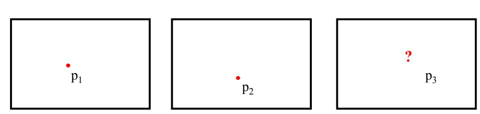


	Linear Least Squares	[Hartley, 1995]	[Luong *et al.*, 1993]
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel



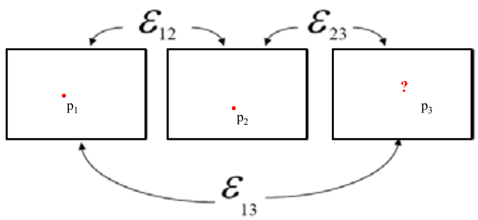
Stereo Constraints

Given p_1, p_2, in the left and right image where is p_3?



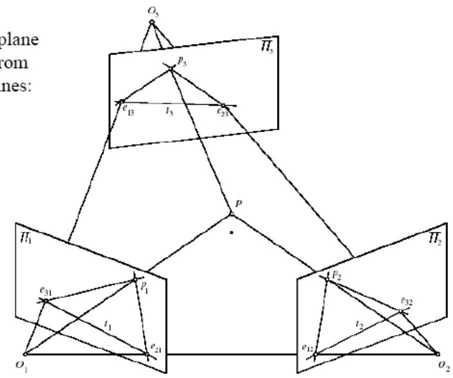
Three Essential Matrices

Essential matrices relate each pair:
(calibrated case)

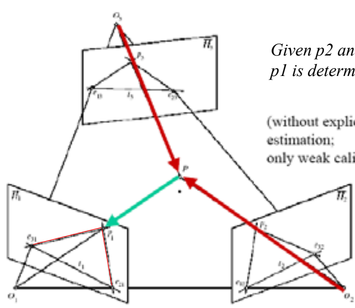


Trinocular Epipolar Geometry

Trifocal plane formed from trifocal lines:



Trinocular Epipolar Geometry



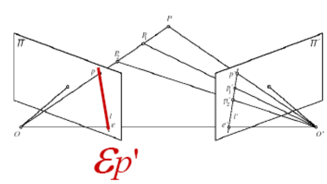
Given p2 and p2, p1 is determined!

(without explicit depth estimation; only weak calibration)

p_1 is at the intersection of the epipolar lines associated with p_2 and p_3

Recall: Epipolar Line Constraint

$\mathcal{E}p'$ is the epipolar line corresponding to p' in the left camera.



$$au + bv + c = 0$$

$$p = (u, v, 1)^T$$

$$l = (a, b, c)^T$$

$$l \cdot p = 0$$

$$\boldsymbol{p}^T \boldsymbol{l} = 0$$

$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0$$

$$\mathcal{E}p' \cdot p = 0$$

Three Essential Matrices

$$\begin{cases} \boldsymbol{p}_1^T \mathcal{E}_{12} \boldsymbol{p}_2 = 0, \\ \boldsymbol{p}_2^T \mathcal{E}_{23} \boldsymbol{p}_3 = 0, \\ \boldsymbol{p}_3^T \mathcal{E}_{31} \boldsymbol{p}_1 = 0, \end{cases}$$

$$\boldsymbol{p}_1^T \mathcal{E}_{12} \boldsymbol{p}_2 = 0$$

$$\boldsymbol{p}_3^T \mathcal{E}_{31} \boldsymbol{p}_1 = 0$$

Combining extrinsic and intrinsic calibration parameters

From lecture 3:

$$\vec{p} = \frac{1}{z} \begin{pmatrix} K & \vec{0} \end{pmatrix} {}^C\vec{P} \quad \text{Intrinsic}$$

$$\begin{pmatrix} {}^C\vec{P} \end{pmatrix} = \begin{pmatrix} {}^C_W R & {}^C O_W \\ 0 \; 0 \; 0 & 1 \end{pmatrix} \begin{pmatrix} \vec{P} \end{pmatrix} \quad \text{Extrinsic}$$

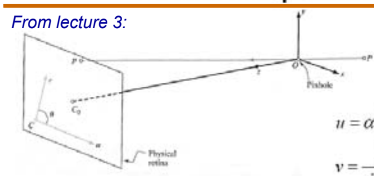
$$\vec{p} = \frac{1}{z} K \begin{pmatrix} {}^C_W R & {}^C O_W \end{pmatrix} \vec{P}$$

$$\vec{p} = \frac{1}{z} \mathcal{M} \vec{P}$$

Forsyth&Ponce

Intrinsic parameters

From lecture 3:



$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$

$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$

Using homogenous coordinates, we can write this as:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \frac{1}{z} \begin{pmatrix} \alpha & -\alpha\cot(\theta) & u_0 & 0 \\ 0 & \frac{\beta}{\sin(\theta)} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

or:

$$\vec{p} = \frac{1}{z} \begin{pmatrix} K & \vec{0} \end{pmatrix} {}^{C}\vec{P}$$

Extrinsic parameters: translation and rotation of camera frame

From lecture 3:

Non-homogeneous coordinates

$${}^{B}P = {}^{B}_{A}R \; {}^{A}P + {}^{B}O_A$$

Homogeneous coordinates

$${}^{B}P = {}^{B}_{A}C \; {}^{A}P$$

where

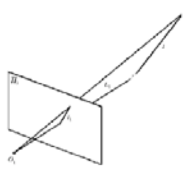
$$C = \begin{pmatrix} {}^{B}_{A}R & {}^{B}O_A \\ 0 \;\; 0 \;\; 0 & 1 \end{pmatrix}$$

Block matrix form

$$\begin{pmatrix} {}^{C}P \\ 1 \end{pmatrix} = \begin{pmatrix} {}^{C}_{W}\mathcal{R} & {}^{C}O_W \\ \mathbf{0}^T & 1 \end{pmatrix} \begin{pmatrix} {}^{W}P \\ 1 \end{pmatrix}$$

Trifocal Line Constraint

Form the plane containing a line l and optical center of one camera:



$$\boldsymbol{l}^T \boldsymbol{p} = 0,$$

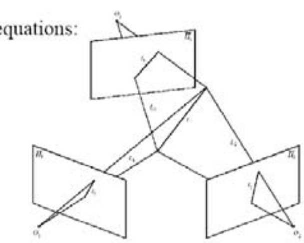
$$\boldsymbol{l}^T \mathcal{M}\boldsymbol{P} = 0,$$

Trifocal Line Constraint

3 cameras, 3 plane equations:

$$\begin{pmatrix} \boldsymbol{l}_1^T \mathcal{M}_1 \\ \boldsymbol{l}_2^T \mathcal{M}_2 \\ \boldsymbol{l}_3^T \mathcal{M}_3 \end{pmatrix} \boldsymbol{P} = \mathbf{0}$$

$$\mathcal{L} \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{l}_1^T \mathcal{M}_1 \\ \boldsymbol{l}_2^T \mathcal{M}_2 \\ \boldsymbol{l}_3^T \mathcal{M}_3 \end{pmatrix}$$



If 3 lines intersect in more than one point (a line) this system is degenerate and is rank 2.

Trifocal Line Constraint

Assume calibrated camera coordinates

$$\mathcal{M}_1 = (\,\mathrm{Id} \quad \mathbf{0}\,)$$

$$\mathcal{M}_2 = (\,\mathcal{R}_2 \quad \boldsymbol{t}_2\,)$$

$$\mathcal{M}_3 = (\,\mathcal{R}_3 \quad \boldsymbol{t}_3\,)$$

then

$$\mathcal{L} = \begin{pmatrix} \boldsymbol{l}_1^T & 0 \\ \boldsymbol{l}_2^T \mathcal{R}_2 & \boldsymbol{l}_2^T \boldsymbol{t}_2 \\ \boldsymbol{l}_3^T \mathcal{R}_3 & \boldsymbol{l}_3^T \boldsymbol{t}_3 \end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix} \boldsymbol{l}_1^T & 0 \\ \boldsymbol{l}_2^T \mathcal{R}_2 & \boldsymbol{l}_2^T \boldsymbol{t}_2 \\ \boldsymbol{l}_3^T \mathcal{R}_3 & \boldsymbol{l}_3^T \boldsymbol{t}_3 \end{pmatrix}$$

Rank $\mathcal{L} = 2$ means det. of 3x3 minors are zero, and can be expressed as:

$$\boldsymbol{l}_1 \times \begin{pmatrix} \boldsymbol{l}_2^T \mathcal{G}_1^1 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^2 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^3 \boldsymbol{l}_3 \end{pmatrix} = \mathbf{0},$$

with

$$\mathcal{G}_1^i = \boldsymbol{t}_2 \boldsymbol{R}_3^{iT} - \boldsymbol{R}_2^i \boldsymbol{t}_3^T$$

The Trifocal Tensor

These 3 3x3 matrices are called the trifocal tensor.

$$\mathcal{G}_1^i = \boldsymbol{t}_2 \boldsymbol{R}_3^{iT} - \boldsymbol{R}_2^i \boldsymbol{t}_3^T$$

the constraint

$$\boldsymbol{l}_1 \times \begin{pmatrix} \boldsymbol{l}_2^T \mathcal{G}_1^1 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^2 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^3 \boldsymbol{l}_3 \end{pmatrix} = \mathbf{0},$$

can be used for point or line transfer.

Trifocal Line Constraint

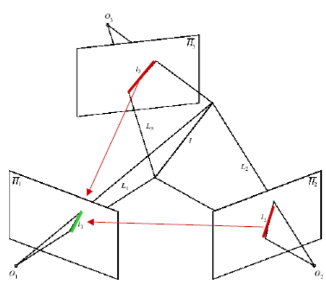
line transfer:

$$\boldsymbol{l}_1 \approx \begin{pmatrix} \boldsymbol{l}_2^T \mathcal{G}_1^1 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^2 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^3 \boldsymbol{l}_3 \end{pmatrix}$$

point transfer via lines: form independent pairs of lines through p2,p3, solve for p1.

Line Transfer

$$\boldsymbol{l}_1 \approx \begin{pmatrix} \boldsymbol{l}_2^T \mathcal{G}_1^1 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^2 \boldsymbol{l}_3 \\ \boldsymbol{l}_2^T \mathcal{G}_1^3 \boldsymbol{l}_3 \end{pmatrix}$$



Uncalibrated Case

$$\mathcal{L} = \begin{pmatrix} \boldsymbol{l}_1^T \mathcal{K}_1 & 0 \\ \boldsymbol{l}_2^T \mathcal{K}_2 \mathcal{R}_2 & \boldsymbol{l}_2^T \mathcal{K}_2 \boldsymbol{t}_2 \\ \boldsymbol{l}_3^T \mathcal{K}_3 \mathcal{R}_3 & \boldsymbol{l}_3^T \mathcal{K}_3 \boldsymbol{t}_3 \end{pmatrix}$$

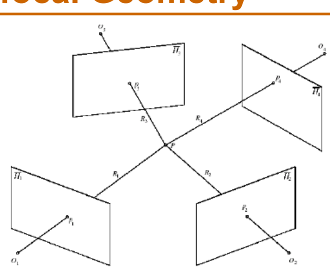
$$\mathcal{A}_i \stackrel{\text{def}}{=} \mathcal{K}_i \mathcal{R}_i \mathcal{K}_1^{-1} \qquad \boldsymbol{a}_i \stackrel{\text{def}}{=} \mathcal{K}_i \boldsymbol{t}_i$$

$$\mathcal{M}_1 = (\mathcal{K}_1 \quad \mathbf{0}),\ \mathcal{M}_2 = (\mathcal{A}_2 \mathcal{K}_1 \quad \boldsymbol{a}_2),$$

$$\mathcal{M}_3 = (\mathcal{A}_3 \mathcal{K}_1 \quad \boldsymbol{a}_3)$$

$$\text{Rank}(\boldsymbol{\mathcal{L}}) = 2 \Longleftrightarrow \text{Rank}(\boldsymbol{\mathcal{L}} \begin{pmatrix} \mathcal{K}_1^{-1} & 0 \\ 0 & 1 \end{pmatrix}) = \text{Rank} \begin{pmatrix} \boldsymbol{l}_1^T & 0 \\ \boldsymbol{l}_2^T \mathcal{A}_2 & \boldsymbol{l}_2^T \boldsymbol{a}_2 \\ \boldsymbol{l}_3^T \mathcal{A}_3 & \boldsymbol{l}_3^T \boldsymbol{a}_3 \end{pmatrix} = 2$$

Quadrifocal Geometry



Can form a "quadrifocal tensor"

Faugeras and Mourrain (1995) have shown that it is algebraically dependent on associated essential/fundamental matricies and trifocal tensor: no new constraints added.

No additional independent constraints from more than 3 views.

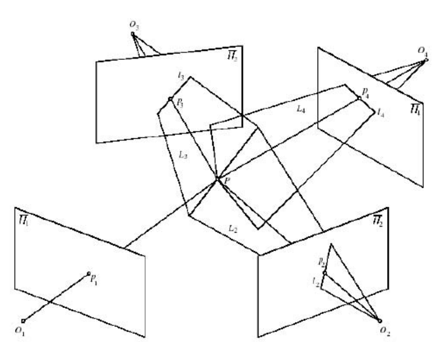


Figure 12.10. Given four images p_1, p_2, p_3 and p_4 of some point P and three arbitrary image lines l_2, l_3 and l_4 passing through the points p_2, p_3 and p_4, the ray passing through O_1 and p_1 must also pass through the point where the three planes L_2, L_3 and L_4 formed by the preimages of these lines intersect.

Trifocal Constraint with Noise

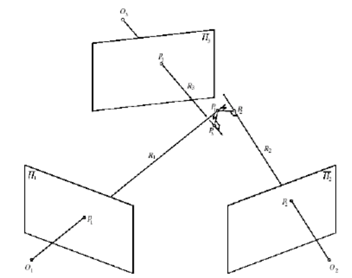


Figure 12.11. Trinocular constraints in the presence of calibration or measurement errors: the rays R_1, R_2 and R_3 may not intersect.

Project

- **Final project may be:**
  - An original implementation of a new or published idea
- **A project proposal not longer than two pages must be submitted by April 1.**

Project

March 24: - Project Previews / Brainstorming

3-5 minute presentation describing

- Specific Project Idea
- Your research, or thesis proposal (if it relates to vision)
- Paper you are interested in and may form the basis of a project